

# $\tan\beta$ determination from the Higgs boson decay at the International Linear Collider\*

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We study the methods and their accuracies for determining  $\tan\beta$  in two Higgs doublet models at future lepton colliders. In addition to the previously proposed methods using direct production of additional Higgs bosons, we propose a method using the precision measurement of the decay branching ratio of the standard-model (SM)-like Higgs boson. The method is available if there is a deviation from the SM in the coupling constants of the Higgs boson with the weak gauge bosons. We find that, depending on the type of Yukawa interactions, this method can give the best sensitivity in a wide range of  $\tan\beta$ .

## I. INTRODUCTION

A Higgs boson has been discovered at the LHC [2, 3]. No indication has been found so far for the deviation from the standard model (SM) on the nature of the Higgs boson such as the decay width, spin-parity, and also on the coupling constants with SM fermions up to current experimental accuracies [4–7]. In spite of such situation, Higgs sector is not yet settled to be composed with one Higgs doublet. Namely, there are various other possibilities which are still consistent with current experimental data, and even more, plenty of models with the extended Higgs sector have been proposed to explain the phenomena which may indicate physics with a new energy scale beyond the SM, such as hierarchy problem, neutrino masses, dark matter, etc.

Searches for the evidence of extended Higgs sectors are of primary importance at future experiments. At the second stage of the LHC experiment with  $\sqrt{s} = 13$  or 14 TeV, direct searches for the evidence of additional Higgs bosons can be performed and the energy reach for new particles will be extended. On the other hand, precise measurements of the couplings of the Higgs boson can be performed at the future International Linear Collider (ILC) experiment [8, 9], and the evidence of non-standard Higgs models can be detected as a deviation from the SM in the coupling constants of the SM-like Higgs boson. Furthermore, some parameter regions where the direct searches at the LHC cannot be covered can be complemented by the searches at the ILC [10]. The model discrimination can be performed through the direct measurement of the properties of additional Higgs bosons and/or fingerprinting the pattern of the deviation in various coupling measurements [11].

In this talk, we discuss collider methods for the determination of  $\tan\beta$ , a ratio of vacuum expectation values of the two doublets, in the two Higgs doublet model (THDM) as a benchmark model for the extended Higgs sector. We propose a new method through the measurements of the branching ratio of the SM-like Higgs boson at future lepton colliders. The method is applicable as long as there exist deviations in the couplings of the SM-like Higgs boson to gauge bosons, even for the case where additional Higgs bosons are too heavy to be detected directly. We studies the sensitivity of determining  $\tan\beta$  at the ILC, and compare it with those for the previously proposed methods which utilize direct production of additional Higgs bosons [12].

## II. TWO HIGGS DOUBLET MODEL

In this section, we briefly review the THDM with a softly-broken discrete  $Z_2$  symmetry. This model has two preferable features which are good to naturally avoid phenomenological constraints on the extended Higgs sector. One is that any multi-doublet model predicts the electroweak rho parameter,  $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ , to be unity at the tree level. Since the experimental constraint on the rho parameter is quite strict,  $\rho_{\text{exp}} = 1.0004^{+0.0003}_{-0.0004}$  [13], these models may be regarded as natural extension of the SM. Second is that the  $Z_2$  symmetry can suppress the flavor changing neutral currents (FCNCs) which are also severely constrained by flavor experiments. Under the  $Z_2$  symmetry, each fermion couples to only one Higgs field, so that the Higgs-mediated FCNCs are prevented at the tree level and the constraints are relaxed to the loop level [14].

In the THDM with  $Z_2$  symmetry, depending on the assignment of the  $Z_2$  parity to each fermion, four types of Yukawa interaction can be constructed [15–17]. Among the four types, we focus on the so-called Type-II and

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\* The talk is based on Ref. [1].

	$\xi_h^u$	$\xi_h^d$	$\xi_h^\ell$	$\xi_H^u$	$\xi_H^d$	$\xi_H^\ell$	$\xi_A^u$	$\xi_A^d$	$\xi_A^\ell$
Type-I	$c_\alpha/s_\beta$	$c_\alpha/s_\beta$	$c_\alpha/s_\beta$	$s_\alpha/s_\beta$	$s_\alpha/s_\beta$	$s_\alpha/s_\beta$	$\cot\beta$	$-\cot\beta$	$-\cot\beta$
Type-II	$c_\alpha/s_\beta$	$-s_\alpha/c_\beta$	$-s_\alpha/c_\beta$	$s_\alpha/s_\beta$	$c_\alpha/c_\beta$	$c_\alpha/c_\beta$	$\cot\beta$	$\tan\beta$	$\tan\beta$
Type-X	$c_\alpha/s_\beta$	$c_\alpha/s_\beta$	$-s_\alpha/c_\beta$	$s_\alpha/s_\beta$	$s_\alpha/s_\beta$	$c_\alpha/c_\beta$	$\cot\beta$	$-\cot\beta$	$\tan\beta$
Type-Y	$c_\alpha/s_\beta$	$-s_\alpha/c_\beta$	$c_\alpha/s_\beta$	$s_\alpha/s_\beta$	$c_\alpha/c_\beta$	$s_\alpha/s_\beta$	$\cot\beta$	$\tan\beta$	$-\cot\beta$

TABLE I: The scaling factors for the four types of Yukawa interactions in the THDM [17].

Type-X (lepton specific) THDMs, since these deserve much interests from the viewpoint of constructing models for physics beyond the SM. Type-II THDM is well-known as the Higgs sector in the minimal supersymmetric extension of the SM, where up-type quarks couple to one Higgs doublet while down-type quarks and charged leptons couple to another Higgs doublet. Type-X THDM is sometimes employed in models for neutrino masses, etc., where quarks couple to one Higgs doublet while charged leptons couple to another Higgs doublet.

For simplicity, we restrict ourselves to consider the CP-conserving scenario, where CP-even  $H$ , CP-odd  $A$  and charged  $H^\pm$  Higgs bosons appear as mass eigenstates in addition to the light CP-even  $h$  which we assume the observed Higgs boson with  $m_h = 125$  GeV. Mixing angles  $\alpha$  and  $\beta$  are defined, respectively, as the angles in the neutral CP-even states and that between  $A$  and  $z$  the neutral component of Nambu-Goldstone boson, as well as between  $H^\pm$  and  $w^\pm$  the charged components of Nambu-Goldstone bosons.  $\beta$  satisfies  $\tan\beta = v_2/v_1$  where  $v_i$  are vacuum expectation values of the two Higgs fields.

Coupling constants of  $\Phi VV$  interactions, where  $\Phi = H$  or  $h$ , are given as

$$g_{hVV}^{\text{THDM}} = g_{hVV}^{\text{SM}} \cdot \sin(\beta - \alpha), \quad g_{HVV}^{\text{THDM}} = g_{hVV}^{\text{SM}} \cdot \cos(\beta - \alpha), \quad (1)$$

where  $g_{hVV}^{\text{SM}}$  is the corresponding coupling constant for the SM Higgs boson. When  $\sin(\beta - \alpha) = 1$ , which is called ‘‘SM-like limit’’ [18],  $h$  has the same coupling constants with gauge bosons as those of the SM Higgs boson. In general,  $\sin(\beta - \alpha)$  is a free parameter in the model. However, a large deviation of  $\sin(\beta - \alpha)$  from unity is restricted by theoretical constraints which are derived by using the argument of perturbative unitarity [19]. Experimental constraints have also been obtained at the LHC [4, 20–22].

The Yukawa couplings for each type in the THDM are characterized by a scaling factor,  $\xi_\phi^f$ , defined as the coupling constant in each model divided by that for the coupling constant in the SM. For example,

$$\xi_h^f = \sin(\beta - \alpha) + \cot\beta \cdot \cos(\beta - \alpha), \quad (2)$$

$$\xi_H^f = \cos(\beta - \alpha) - \cot\beta \cdot \sin(\beta - \alpha), \quad (3)$$

for  $f = u$  in Type-II and  $f = u, d$  in Type-X, while

$$\xi_h^f = \sin(\beta - \alpha) - \tan\beta \cdot \cos(\beta - \alpha), \quad (4)$$

$$\xi_H^f = \cos(\beta - \alpha) + \tan\beta \cdot \sin(\beta - \alpha), \quad (5)$$

for  $f = d, \ell$  in Type-II and  $f = \ell$  in Type-X. Thus, in the SM-like limit, Yukawa couplings for  $h$  become the same as those in the SM, and their  $\tan\beta$  dependence disappears. On the other hand,  $\tan\beta$  dependence on the Yukawa couplings for  $H$ , as well as  $A$  and  $H^\pm$ , remains in this limit. The scaling factors are summarized in Table. I for the four types of Yukawa interaction in the THDM.

In Fig. 1, we show branching ratios of neutral Higgs bosons in the  $b\bar{b}$  and  $\tau^+\tau^-$  decay modes for  $m_h = 125$  GeV and  $m_H = m_A = 200$  GeV in the Type-II and Type-X THDM. From the left,  $\mathcal{B}(b\bar{b})$  for Type-II with  $\sin^2(\beta - \alpha) = 1$ , that with  $\sin^2(\beta - \alpha) = 0.99$ ,  $\mathcal{B}(\tau^+\tau^-)$  for Type-X with  $\sin^2(\beta - \alpha) = 1$ , that with  $\sin^2(\beta - \alpha) = 0.99$  are plotted as a function of  $\tan\beta$ , respectively. Solid (dashed) lines are for  $\cos(\beta - \alpha) < 0$  ( $\cos(\beta - \alpha) > 0$ ). We see that, when  $\sin^2(\beta - \alpha) = 1$ , branching ratios of  $h$  are independent of  $\tan\beta$ . However, once it deviates from unity, there appear substantial  $\tan\beta$  dependence, and the branching ratios vary in a wide range.  $\tan\beta$  dependence on the branching ratios of  $H$  and  $A$  is also large, and it remains even in the SM-like limit.

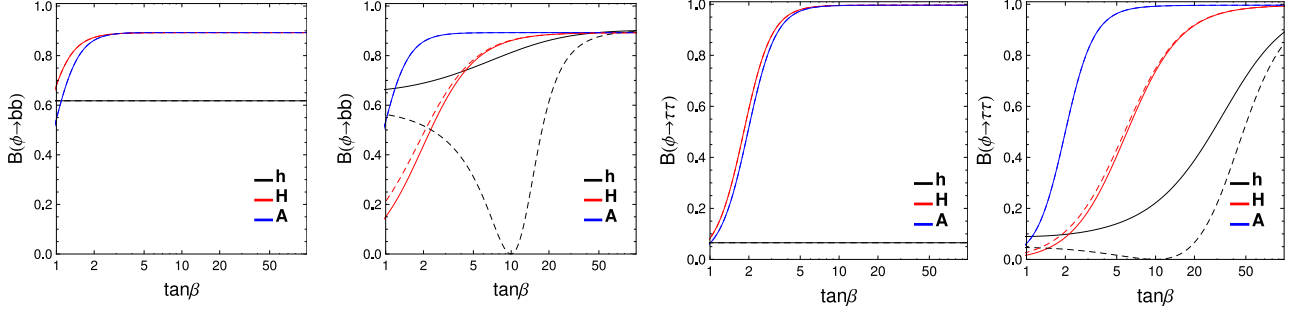


FIG. 1: Left two panels: the decay branching ratios for  $h \rightarrow b\bar{b}$  (black curves),  $H \rightarrow b\bar{b}$  (red curves), and  $A \rightarrow b\bar{b}$  (blue curves) decays as a function of  $\tan\beta$  in the Type-II THDM with  $\sin^2(\beta - \alpha) = 1$  and  $0.99$ , respectively. The solid (dashed) curves denote the case with  $\cos(\beta - \alpha) \leq 0$  ( $\cos(\beta - \alpha) \geq 0$ ). Right two panels: the same as left two panels, but for the  $\tau^+\tau^-$  decays in the Type-X THDM.

### III. $\tan\beta$ MEASUREMENT

In this section, we discuss methods of  $\tan\beta$  determination in the future lepton colliders. We consider three methods which utilize the measurements of the following observables, respectively,<sup>1</sup> (i) branching ratios of  $H$  and  $A$ ,  $\mathcal{B}_{H,A}$ , (ii) total decay widths of  $H$  and  $A$ ,  $\Gamma_{H,A}$ , (iii) branching ratio of  $h$ ,  $\mathcal{B}_h$ . The first two observables can be studied in the direct production of  $H$  and  $A$ , i.e., the  $e^+e^- \rightarrow HA$  process. Thus, these can be available if the sum of the mass of  $H$  and  $A$  is less than the collider energy. Since the production cross section is independent of model parameters, and the branching ratio of  $H$  and  $A$  in the  $b\bar{b}$  and  $\tau^+\tau^-$  decay modes significantly depend on  $\tan\beta$ ,  $\tan\beta$  can be determined by counting the number of events for  $4b$  ( $4\tau$ ) events;  $\mathcal{N} \propto \sigma_{HA} \cdot \mathcal{B}_H \cdot \mathcal{B}_A$ . Thus, the observation of the branching ratios gives  $\tan\beta$  determination by comparing with the theoretical prediction. We note that masses of  $H$  and  $A$  can be easily measured from the peak in the invariant mass distribution.

The last method utilizes the precision measurement of the branching ratio of  $h$ . In the THDM, as we see in Eqs. (2-5), when  $\sin(\beta - \alpha) < 1$ , the Yukawa couplings of  $h$  can be deviated from those in the SM. It is known that the pattern of the deviations for up-type, down-type quarks and charged leptons depends on the type of Yukawa interaction, therefore, by observing it we could distinguish the type of Yukawa interaction in the THDM [11, 23]. Furthermore, the magnitude of the deviation depends on the value of  $\tan\beta$ , so that we can determine  $\tan\beta$  by observing it. The accuracy of the  $\tan\beta$  determination depends on how accurately the branching ratio can be measured experimentally and also how steeply the branching ratio depends on  $\tan\beta$ .

### IV. RESULTS

In this section, we study the accuracies of  $\tan\beta$  measurement for the above three methods at the ILC.

For the method (i), the sensitivity is estimated as follows. We utilize  $b\bar{b}$  decay mode for Type-II and  $\tau^+\tau^-$  decay mode for Type-X which are basically large and also which have large  $\tan\beta$  dependence. The expected number of events for  $4b$  and  $4\tau$  events can be obtained as  $N = \sigma_{HA} \cdot \mathcal{B}_H \cdot \mathcal{B}_A \cdot \mathcal{L} \cdot \epsilon$ , where  $\epsilon$  is the acceptance ratio for observing  $4b$  and  $4\tau$  signals. We take  $m_H = m_A = 200$  GeV and  $\sqrt{s} = 500$  GeV with  $\mathcal{L} = 250$  fb<sup>-1</sup>.  $\epsilon_{4b}$  and  $\epsilon_{4\tau}$  are estimated to be 50% for both by our simulation [1]. The  $1\sigma$  sensitivity to  $\tan\beta$  is obtained by solving  $N(\tan\beta \pm \Delta\tan\beta) = N_{\text{obs}} \pm \Delta N_{\text{obs}}$ , where  $\Delta N_{\text{obs}} = \sqrt{N_{\text{obs}}}$  is a statistical error.

For the method (ii),  $\tan\beta$  sensitivity from the width measurement is estimated as follows. The detector resolutions for the Breit-Wigner width in the  $b\bar{b}$  and  $\tau^+\tau^-$  invariant mass distributions are estimated to be  $\Gamma_{b\bar{b}}^{\text{res}} = 11$  GeV and  $\Gamma_{\tau^+\tau^-}^{\text{res}} = 7$  GeV, respectively [1]. The width to be observed is

$$\Gamma_{H/A}^R = \frac{1}{2} \left[ \sqrt{(\Gamma_H^{\text{tot}})^2 + (\Gamma^{\text{res}})^2} + \sqrt{(\Gamma_A^{\text{tot}})^2 + (\Gamma^{\text{res}})^2} \right], \quad (6)$$

<sup>1</sup> The other method by using the cross-section measurements of  $b\bar{b}H + b\bar{b}A$  production is also proposed in Ref. [12]. This method may become useful for heavier  $H$  and  $A$  case where  $H$  and  $A$  cannot be produced in pair but only singly due to the kinematical limitation.

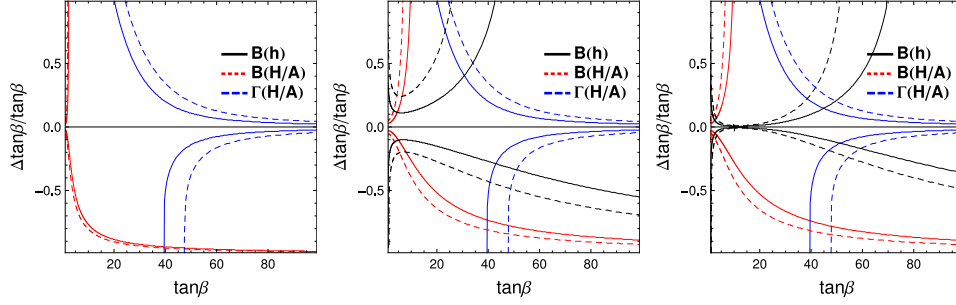


FIG. 2: Sensitivities to the  $\tan\beta$  measurement in the Type-II THDM. From the left,  $\sin(\beta - \alpha) = 1$ ,  $\sin^2(\beta - \alpha) = 0.99$  with  $\cos(\beta - \alpha) < 0$ , and  $\sin^2(\beta - \alpha) = 0.99$  with  $\cos(\beta - \alpha) > 0$  are taken, respectively.

and the  $1\sigma$  uncertainty is given by [12]

$$\Delta\Gamma_{H/A}^R = \sqrt{\left(\Gamma_{H/A}^R / \sqrt{2N_{\text{obs}}}\right)^2 + \left(\Delta\Gamma_{\text{sys}}^{\text{res}}\right)^2}, \quad (7)$$

where  $\Delta\Gamma_{\text{sys}}^{\text{res}}$  is taken as 10% of  $\Gamma^{\text{res}}$  for each decay mode. Then,  $1\sigma$  sensitivity for the  $\tan\beta$  determination is obtained by solving  $\Gamma_{H/A}(\tan\beta \pm \Delta\tan\beta) = \Gamma_{H/A}^R \pm \Delta\Gamma_{H/A}^R$ , where  $\Gamma_{H/A} = \frac{1}{2}(\Gamma_H + \Gamma_A)$ .

For the method (iii),  $\tan\beta$  sensitivity is evaluated by solving  $\mathcal{B}_h(\tan\beta \pm \Delta\tan\beta) = \mathcal{B}_h^{\text{obs}} \pm \Delta\mathcal{B}_h^{\text{obs}}$ , where the accuracy of  $\mathcal{B}_h$  measurement is evaluated from the reference value by rescaling the statistical factor by taking into account the change of the expected number of events. The reference values for the  $1\sigma$  accuracy of determining branching ratios in the  $b\bar{b}$  and  $\tau^+\tau^-$  decay modes at the ILC with  $\sqrt{s} = 250$  GeV and  $\mathcal{L} = 250 \text{ fb}^{-1}$  are taken as 1.3% and 2%, respectively, from the recent reports [8, 9]<sup>2</sup>.

In Fig. 2, our numerical results for the three methods are shown for the Type-II THDM. Left panel is for  $\sin^2(\beta - \alpha) = 1$ , middle panel is for  $\sin^2(\beta - \alpha) = 0.99$  with  $\cos(\beta - \alpha) < 0$ , and right panel is for  $\sin^2(\beta - \alpha) = 0.99$  with  $\cos(\beta - \alpha) > 0$ .  $1\sigma$  (solid) and also  $2\sigma$  (dashed) sensitivities are drawn as a function of  $\tan\beta$  for each method. In the left panel, the method (iii) does not work, since there is no  $\tan\beta$  sensitivity in the SM-like limit. The method (i) has good sensitivity in smaller  $\tan\beta$  regions, since there exists  $\tan\beta$  dependence in  $\mathcal{B}_{H/A}$  only for these regions. The method (ii) has good sensitivity in larger  $\tan\beta$  regions, where the widths can be directly measured. In the middle and right panels, when  $\sin^2(\beta - \alpha) < 1$ , the method (iii) works very well in wide regions in  $\tan\beta$ .

In Fig. 3, numerical results for the three methods are also shown for the Type-X THDM in the same manner as Fig. 2. The features of the three methods are similar to those for Type-II.

## V. SUMMARY

We have studied the sensitivities of  $\tan\beta$  measurement by using the complementary three methods at the ILC; (i) the branching ratio of  $H$  and  $A$ , (ii) the total decay width of  $H$  and  $A$ , and (iii) the branching ratio of  $h$ . The first two methods utilize the direct observation of the additional Higgs bosons,  $H$  and  $A$ . Therefore, these methods are available if the production process of  $e^+e^- \rightarrow HA$  is kinematically accessible. The last method utilizes the precision measurement of the branching ratio of  $h$  at the ILC. Although the method is available only for the case with  $\sin(\beta - \alpha) < 1$  where  $\tan\beta$  dependence can be seen in the branching ratio of  $h$ , the method has better sensitivity for determining  $\tan\beta$  than the other methods in a wide range of  $\tan\beta$ .

<sup>2</sup> We note that an indication of  $\sin(\beta - \alpha) \neq 1$  in the THDM can be obtained by measuring the absolute value of the  $hVV$  couplings at the ILC. At the ILC with  $\sqrt{s} = 250$  GeV and  $\mathcal{L} = 250 \text{ fb}^{-1}$ , the best accuracy of the measurements can be expected for the  $hZZ$  coupling at 0.7% [8, 9].

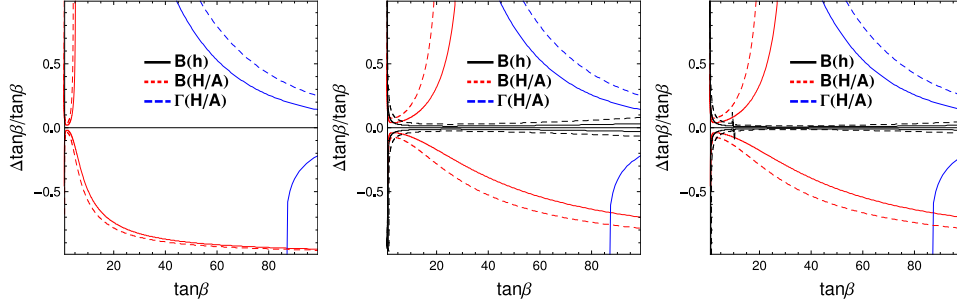


FIG. 3: The same as Fig. 2, but for the Type-X THDM.

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